Radiatively-Induced Lorentz and CPT Violating Chern-Simons Term in QED

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Abstract

We calculate the induced Lorentz- and CPT-violating Chern-Simons term arising from the Lorentz- and CPT-violating sector of quantum electrodynamics with a $b_\mu \bar{\psi} \gamma^\mu \gamma_5 \psi$ term. The result to all orders in b coincides with the previous linear-in-b calulation by Chung and Oh [hep-th/9812132] as well as Jackiw and Kostelecký [Phys. Rev. Lett. **82**, 3572 (1999)], since all higher order terms vanish.

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Recently, Colladay and Kostelecký [1] posed the question whether a Lorentz- and CPT-violating Chern-Simons term [2] is induced when the Lorentz- and CPT-violating term $\bar{\psi}\not{b}\gamma_5\psi$ (b_μ a constant 4-vector) is added to the conventional Lagrangian of QED [3]. This has been discussed by a number of authors [4–9]. In Refs. [1,4–7], only the term linear in b_μ was kept in the calculation of the vacuum polarization (which determines the induced Chern-Simons term). A calculation of higher order (in b) corrections was attempted by Fosco and Le Guillou [8]. However, their non-trivial result is invalid because they use an incorrect fermion propagator. After completion of our exact, i.e., non-perturbative in b_μ, calculation, we received a paper by Pérez-Victoria [9], in which the same calculation was carried out, with the same result as we report here: for $b < m$ all higher order terms vanish.

Consider a Dirac fermion propagating in a Lorentz- and CPT-violating manner in the background of a photon field. The Lagrangian of this system is

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi - \bar{\psi}\not{b}\gamma_5\psi - \bar{\psi}\not{A}\psi \ , \tag{1}$$

where b_μ is a constant 4-vector, and $b_\mu\bar{\psi}\gamma^\mu\gamma_5\psi$ is the Lorentz violating, CPT-odd term. The fermion propagator is

$$G(p) = \frac{i}{\not{p} - m - \not{b}\gamma_5} \ . \tag{2}$$

If we observe the following relation

$$\begin{aligned}(\not{p} - m - \not{b}\gamma_5)(\not{p} + m - \not{b}\gamma_5)(\not{p} + m + \not{b}\gamma_5)(\not{p} - m + \not{b}\gamma_5)& \\ = (p^2 - m^2 - b^2)^2 - 4(p\cdot b)^2 + 4p^2b^2 \ ,&\end{aligned}$$

we can readily rationalize the propagator, Eq. (2), as follows:

$$G = \frac{i}{\not{p} - m - \not{b}\gamma_5} = \frac{i(U + V\gamma_5)}{D} \ , \tag{3}$$

where

$$\begin{aligned}U &= (p^2 - m^2 + b^2)(\not{p} + m) - 2mb^2 - 2p\cdot b\not{b} \ , \\ V &= (p^2 - m^2 + b^2)\not{b} - (2p\cdot b + 2m\not{b})(\not{p} - m) \ , \\ D &= (p^2 - m^2 - b^2)^2 - 4(p\cdot b)^2 + 4p^2b^2 \ .\end{aligned}$$

The vacuum polarization tensor is given by

$$\Pi^{\mu\nu}(p) = \int \frac{d^4k}{(2\pi)^4}\mathrm{tr}[\gamma^\mu G(p+k)\gamma^\nu G(k)] \ , \tag{4}$$

and its antisymmetric part has the following general structure:

$$\begin{aligned}\Pi^{[\mu\nu]} &\equiv \frac{1}{2}(\Pi^{\mu\nu} - \Pi^{\nu\mu}) \\ &= \epsilon^{\mu\nu\alpha\beta}p_\alpha b_\beta\Pi(p^2, p\cdot b, b^2) + (p^\mu b^\nu - b^\mu p^\nu)F(p^2, p\cdot b, b^2) \ .\end{aligned} \tag{5}$$

The derivative of $\Pi^{[\mu\nu]}$ with respect to the external momentum p_α at $p_\alpha = 0$ gives

$$\frac{\partial}{\partial p_\alpha}\Pi^{[\mu\nu]}\Big|_{p=0} = \epsilon^{\mu\nu\alpha\beta}b_\beta\Pi(0,0,b^2) + (g^{\mu\alpha}b^\nu - g^{\nu\alpha}b^\mu)F(0,0,b^2) \ . \tag{6}$$

The first term on the right-hand side of the above equation produces the induced Chern-Simons term in the effective action as follows:

$$\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}k_\mu A_\nu F_{\alpha\beta} \ , \quad \text{with} \quad k_\mu = -\frac{1}{2}b_\mu\Pi(0,0,b^2) \ . \tag{7}$$

Since our concern here is to obtain the induced Chern-Simon term, we do not consider the second term on the right-hand side of Eq. (6); we only collect terms propotional to $\epsilon^{\mu\nu\alpha\beta}$ from $\frac{\partial}{\partial p_\alpha}\Pi^{[\mu\nu]}|_{p=0}$. The complicated trace algebra contains eight gamma matrices together with γ_5. We perform these trace calculations using the package "Feynpar.m" which runs in the Mathematica System. (If interested readers request the trace calculation program, the author will gladly provide it.) After the trace calculation, we are left with the momentum integral:

$$\epsilon^{\mu\nu\alpha\beta}b_\beta\Pi(0,0,b^2) = -4i\epsilon^{\mu\nu\alpha\beta}b_\beta\int\frac{d^4k}{(2\pi)^4}\frac{R_1 + (k\cdot b)^2R_2 + (k\cdot b)^4R_3}{D^3} \ , \tag{8}$$

where

$$\begin{aligned} R_1 &= -b^8 - 2b^6k^2 + 2b^2k^6 + k^8 - 6b^6m^2 - 4b^4k^2m^2 + 2b^2k^4m^2 \\ &\quad - 12b^4m^4 + 6b^2k^2m^4 - 6k^4m^4 - 10b^2m^6 + 8k^2m^6 - 3m^8 \ , \\ R_2 &= 8b^4 + 4b^2k^2 - 8k^4 - 4k^6/b^2 + 28b^2m^2 - 16k^2m^2 \\ &\quad + 12k^4m^2/b^2 + 24m^4 - 12k^2m^4/b^2 + 4m^6/b^2 \ , \\ R_3 &= -16 + 16k^2/b^2 - 16m^2/b^2 \ . \end{aligned}$$

In order to perform this 4-dimensional Minkowski-momentum integral, we first change the time-component of the k and b vectors, k^0 and b^0, into

$$\begin{aligned} k^0 &\to k^4 = ik^0 \ , \\ b^0 &\to b^4 = ib^0 \ , \end{aligned}$$

to get an Eucledian metric. This is allowed as long as $b^2 < m^2$. Then we use 4-dimensional spherical polar coordinates. Furthermore, with replacements $k^2 = m^2x$ and $b^2 = m^2y$, and an introduction of a parameter ϵ, which is set to zero before the radial k (or x) integration, in the following fashion:

$$\frac{1}{D^3} = \frac{1}{2}\frac{\partial^2}{\partial\epsilon^2}\frac{1}{D+\epsilon}\Big|_{\epsilon\to 0} \ ,$$

the right-hand side of Eq. (8) becomes

$$\begin{aligned} &\frac{\epsilon^{\mu\nu\alpha\beta}b_\beta}{4\pi^3}\frac{\partial^2}{\partial\epsilon^2}\int_0^\infty xdx\int_0^\pi \sin^2\theta d\theta\Big[\frac{A + B\cos^2\theta + C\cos^4\theta}{\alpha\sin^2\theta + \beta + \epsilon}\Big]\Big|_{\epsilon=0} \\ &= \frac{\epsilon^{\mu\nu\alpha\beta}b_\beta}{2\pi^2}\int_0^\infty dx\Big\{\frac{Cx}{\alpha^3} + \Big[\frac{A(\alpha+4\beta)}{(\alpha+\beta)^2} + \frac{B}{\alpha+\beta} + \frac{C(\alpha^2 - 4\alpha\beta - 8\beta^2)}{\alpha^3}\Big]\frac{x}{8\beta\sqrt{\beta(\alpha+\beta)}}\Big\} \ , \end{aligned} \tag{9}$$

where

$$\begin{aligned}
A &= (x-y+1)(x+y-3)(\alpha+\beta)\ , \\
B &= -4x(x-y+1)(\alpha+\beta+y[x+y-3])\ , \\
C &= 16x^2y(x-y+1)\ , \\
\alpha &= 4xy\ , \\
\beta &= (x-y+1)^2\ .
\end{aligned}$$

The second term in the integrand of Eq. (9) may be further simplified as

$$\frac{1}{2}\Big[1+\frac{x}{y}-\frac{\alpha+\beta}{2y^2}-\frac{(5+y)x}{\alpha+\beta}-\frac{(1-y)^2}{\alpha+\beta}\Big]\frac{1}{\sqrt{\alpha+\beta}}\ .$$

The final result of our calculation is

$$\Pi(0,0,b^2) = -\frac{3}{8\pi^2}\ . \tag{10}$$

From Eqs. (10) and (7), we find that the strength of the induced Chern-Simons term is given as

$$k_\mu = \frac{3b_\mu}{16\pi^2}\ . \tag{11}$$

This coincides with the previous result by Chung and Oh [4] as well as by Jackiw and Kostelecký [5], in which the b_μ-linear contribution to the induced Chern-Simons term was calculated. We understand the identity of the lowest-order calculation with the all-order calculation by following an argument of Coleman and Glashow [10], (communicated to us by S. Coleman via R. Jackiw). Consider expanding the b-dependent vacuum polarization amplitude in powers of b (b-perturbation theory). In nth order there is a two-index, i.e. two-photon, amplitude, with n chiral insertions (of $\not{b}\gamma_5$). All except the first order are free of linear divergences. (Abelian axial anomalies come only from the triangle graph! [11]) Hence there is no ambiguity in evaluating the higher order graphs, and the Coleman-Glashow "trick" may be used: momentarily let each of the two photons carry different momenta, say p_1 and p_2 (this means that the chiral insertions carry non-zero momentum); from gauge invariance (transversality) in *each* of the photons, we learn that the amplitude is $O(p_1)$ and $O(p_2)$; i.e. it is $O(p_1p_2)$; now go to equal momenta, $p_1 = p_2 = p$, and observe that the amplitude must be $O(p^2)$. The Chern-Simons term one is seeking is $O(p)$; hence all these higher-order graphs do not contribute. The only contribution comes from the lowest order, which is regularization dependent (or unique if the method of evaluation by Jackiw and Kostelecky [5] is adopted).

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